A Neurodynamic Framework for Generalized Pinball-Loss SVMs: Admissible Smoothings, Provable Convergence, and an MRI Bone-Density Application

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Abstract

We extend the neurodynamical neural network for the generalized pinball-loss support vector machine (GPL–SVM) from a single hyperbolic-tangent smoothing to a *smoothing-function-agnostic* model, the GS–GPIN–SVM. For an entire *admissible class* of smoothings of the plus function we establish existence, uniqueness, and—through a single composite Lyapunov function—global asymptotic and global exponential stability of the equilibrium, together with exponential decay of the observable optimality residual $\|\nabla G(\omega(t))\| = \|\dot{\omega}(t)\| / \kappa$. Within this class we isolate two compactly supported polynomial smoothings—the Zang (C^1) and Ye–Liu–Zhou–Liu (C^2) functions—yielding the CS–GPIN–SVM, which recovers the GPL exactly outside a vanishing band, uses only elementary arithmetic, and delivers true sparsity. We further prove a uniform approximation-error bound and a gradient-consistency theorem: as the smoothing parameter tends to infinity, the network equilibria converge to the global minimiser of the exact non-smooth GPL–SVM. Experiments on synthetic, six UCI, and large-scale benchmarks confirm the theory—the compact smoothings attain the smallest approximation error, the highest sparsity, and the best tuned accuracy rank—and an MRI-based bone-mineral-density feasibility study ($n = 194$) shows a class-balanced RBF variant competitive with strong classical and deep-learning baselines.

Key contributions

- **Unified theory.** One admissibility definition covers global (tanh, sigmoid, erf, ...) and compactly supported smoothings under a single convergence and stability analysis.
- **Composite-Lyapunov stability.** A single Lyapunov function controls the state error *and* the optimality residual, giving global exponential stability for the whole class (including the C^1 Zang smoothing) and a sharper asymptotic rate for the C^2 members.
- **Exactness and sparsity.** Compact smoothings reproduce the generalized pinball loss outside a vanishing band, using only elementary arithmetic and yielding genuinely sparse models.
- **Real-world application.** An MRI-based bone-mineral-density study ($n = 194$) demonstrates clinical feasibility against classical and deep-learning baselines.

Keywords: Generalized pinball loss; Neurodynamic neural networks; Smoothing functions; Gradient consistency; Support vector machines; Lyapunov stability; GS–GPIN–SVM; CS–GPIN–SVM.